COMPARISON OF CALCULATION RESULTS USING 5-, 7-, AND 11-COMPONENT AIR MODELS

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The effects of vibrational relaxation and the number of ionized air components that can be allowed for on the flow characteristics in the viscous shock layer are shown.

Development of promising space vehicles that can maneuver in the upper atmospheric layers of Earth using aerodynamic deceleration [1] evokes interest in studying flows with a low density, high ethalpy, and essentially nonequilibrium effects. The fact that it is currently impossible to make terrestrial models of such regimes with respect to all main parameters as a whole has put numerical studies forward.

The study [2] showed that when the spacecraft "Buran" started moving along the planned trajectory the vibrational relaxation exerted an important influence on the heat flux practically over the entire heat-stressed section of the trajectory. At entry velocities $V_\infty \approx 10$ km/sec ionization proceeded intensively in the shock layer near the body. The present article is aimed at estimating the effects of ionization and nonequilibrium excitation of the vibrational degrees of freedom on flow characteristics near the bodies that move along the rebounding trajectory.

In [2], the system of the equations for a hypersonic viscous shock layer on the deceleration line uses the thermodynamically equilibrium approximations of heat capacities and enthalpies (also including the equilibrium state with respect to electron degrees of freedom). For the conditions considered in the present article the shock layer temperature constitutes $10^3 \leq T \leq 5\cdot 10^4$ K, and the equations are written in another form. Let us give here only the equations for the heat flux, vibrational relaxation, and the Stefan–Maxwell relations with regard to the ionized components

$$\rho c_p^{tr} DT - Dp = \frac{\partial}{\partial y}\left(\lambda^{tr}\frac{\partial T}{\partial y}\right) - \sum_{i=H}\left((h_i^{tr} + h_{i0})\dot{w}_i + c_{pi}^{tr} I_i \frac{\partial T}{\partial y}\right) - \sum_{k=M}(Q_k^{VT} + Q_k^R); \tag{1}$$

$$\rho c^v DT_v = \frac{\partial}{\partial y}\left(\lambda^v \frac{\partial T_v}{\partial y}\right) + \sum_{k=M}\left(Q_k^{VT} + Q_k^R - e_k^v \dot{w}_k - c_k^v I_k \frac{\partial T_v}{\partial y}\right); \tag{2}$$

$$\frac{\partial}{\partial y}(x_i) + x_i x_e\left(\frac{\partial}{\partial y}(\ln T) - \frac{\partial}{\partial y}(\ln T_v)\right) - x_i e_i \frac{E}{kT} = \sum_{j=H}\frac{\rho}{\mu} S_{ij} x_i x_j (V_j - V_i) + \frac{\rho}{\mu} S_{ie} x_i x_e V_e,\ i = 1, \ldots, N-1; \tag{3}$$

$$c_i = \frac{m_i x_i}{m},\ m = \sum_{j=H} m_j x_j,\ \sum_{j=1}^{N} z_j x_j = \sum_{j=1}^{N} z_j x_j V_j = 0,$$

$$p = R_A \rho T \sum_{i=H}\frac{c_i}{m_i}\sum_{i=H} I_i = 0,\ \sum_{i=H} c_i = 1,\ D = v\frac{\partial}{\partial y},$$

$$e_i^v = \frac{R_A}{m_i}\frac{\theta_i}{\exp(\theta_i/T_v) - 1},\ c_i^v = \frac{\partial e_i^v}{\partial T_v},$$

$$c^v = \sum_{i=M} c_i^v c_i,\ \lambda^v = \sum_{j=M} c_j \frac{\mu}{S_j} c_j^v,\ S_j = \sum_{i=H} x_i S_{ij}.$$

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Equation (2) is the result of summing the balance equations of vibrational energy of molecules, assuming the equality of their vibrational temperatures.

The boundary conditions on the body surface, e.g., the conditions for no-flow, adhesion, heterogeneous course of the recombination reactions, the conditions for heat balance, and the thermodynamic equilibrium condition, for a translational temperature are given by

$$u = v = 0,\ I_i = -k_{wi}\rho c_i,\ q = \varepsilon\sigma_B T_w^4,\ T_v = T_w.$$

Here q, ε, k_{wi} are the heat flux, the surface emissivity coefficient, and the catalytic recombination coefficient, respectively; σ_B is the Boltzmann constant; T_w is the equilibrium surface temperature. For the ionized components it is assumed that $k_{wi} = \infty$.

The generalized Rankine–Hugoniot relations are assigned as the boundary conditions on the shock wave.

Stefan–Maxwell relations (3) for a two-temperature mixture (translational temperatures of heavy particles and electrons) are obtained in [3]. In Eq. (3), the translational temperature of electrons is assumed to be equal to the vibrational one of molecules (due to the high efficiency of electron vibrational exchange). Barodiffusion is neglected.

The vibrational relaxation effect on the dissociation reactions manifests itself in varying the dissociation rate constant. In the present article, the model from [4] is used:

$$K^D(T,\ T_v) = K^L(T)\,V(T,\ T_v),\quad V(T,\ T_v) = \frac{Z(T)\,Z(T_F)}{Z(T_v)\,Z(-U)},$$
$$\frac{1}{T_F} = \frac{1}{T_v} - \frac{1}{T} - \frac{1}{U}, \tag{4}$$

where $K^D(T)$ is the dissociation constant under thermodynamic equilibrium when $T_v \equiv T$; Z is the partition sum with respect to the vibrational levels; U is the parameter determining the predominant dissociation degree at the upper-lying levels as opposed to the lower ones. In finding the recombination rate constants it was assumed V = 1. The vibrational relaxation effect on the exchange rates was not allowed for. It was assumed that the ionization rates, with the electrons participating in the ionization process, were controlled by the vibrational temperature and, with no participation, including the re-charging reactions, by the translational temperature.

The reverse effect of the chemical reactions on vibrational relaxation was taken into account in terms of the source term Q^R_k [5]. For example, for the O_2 molecules we have

$$Q_k^R = -m_k\left(\frac{\rho}{m}\right)^2\left[\left(\sum_{i=1}^{N} K_{1i}x_i\right)\left(e_{Dk}\frac{K_{pi}}{p}V_i x_k - e_{Rk}x_O^2\right) + K_2 e_k\left(K_{p2}x_O x_{NO} - x_N x_k\right)\right],\quad k = O_2,\quad x_i = \frac{mc_i}{m_i},$$

$$e_{Dk} = e_k(T_{Fk}),\quad e_{Rk} = e_k(-U_k),$$

where K_{1i}, K_{p1} are the rate constants and the equilibrium constant for the oxygen recombination reaction; K_2, K_{p2}, for the exchange reaction with the oxygen participation; e_{Dk} is the mean value of vibrational energy lost by the O_2 molecules due to a single dissociation event; e_{Rk} is the mean value of vibrational energy lost by the O_2 molecules due to a single recombination event.

Energy exchange between the translational and vibrational freedom degrees Q_k^{VT} was calculated in Landau–Teller's form:

$$Q_k^{VT} = \rho c_k\,\frac{e_k^v(T) - e_k^v(T_v)}{\tau_k}.$$

To calculate the vibrational relaxation time τ_k we used the approximations [6] and the correction for high temperatures proposed in [7].

Let us consider flow in the vicinity of the critical line of the body moving along the rebounding trajectory (V_∞ as a function of flight height is shown in Fig. 1). In calculations, the body surface is assumed noncatalytic, R = 6 m, ε = 0.8, $U_k = T_{Dk}/6$ where T_{Dk} is the dissociation temperature of the k-th molecule. The numerical method is similar to the one from [2].

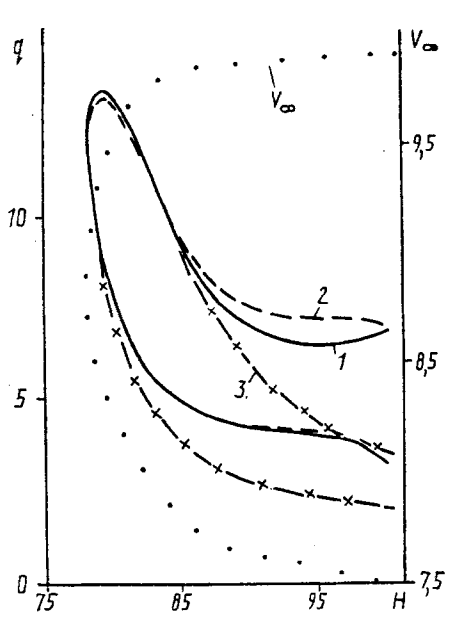

Fig. 1

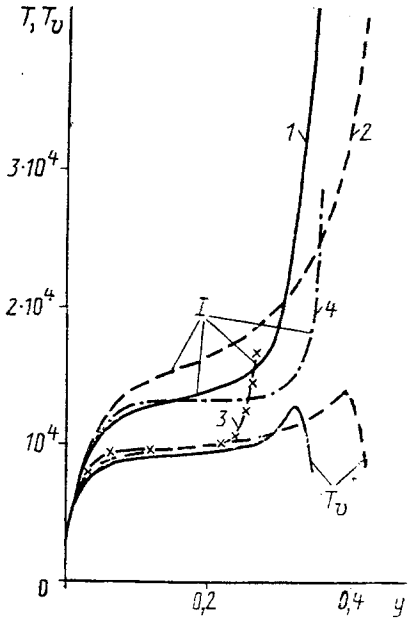

Fig. 2

Fig. 1. Heat flux (1-3) and body velocity vs the flight height: 1, 2) 11, 7-component air models with regard to vibrational relaxation; 3) 11-component model with no regard for vibrational relaxation. q, 10^4 W/m^2; H, km; V_∞, km/sec.

Fig. 2. Translational and vibrational temperature profiles at H = 86 km, V_∞ = 9.8 km/sec; 1-3) notation is the same as in Fig. 1; 4) 7-component model with no regard for vibrational relaxation. T, T_v, K; y, m.

Under the conditions considered 11 components are to a great extent present in the shock layer: N_2, NO, O_2, N, O, N^+_2, NO^+, O^+_2, N^+, O^+, e^-. Between them the reactions of dissociation, recombination, exchange, ionization, and recharging take place. The reaction rate constants used in the present calculations correspond to those in [8]. In addition, we considered the 5-component air model consisting of the neutral particles alone and the 7-component model consisting of the neutral NO^+ and e^- particles. In the case of the 7-component model with the participation of the ionized component there proceeded only one reaction of associative ionization. Use of a more simple air model permited one to effectively reduce the necessary resources and computer time.

Figure 1 plots the heat flux vs the flight height calculated by different air models. The calculations made for the spacecraft "Buran" showed that the ionization degree in the shock layer along the entire planned trajectory did not exceed 0.1%, and it was sufficient to use the 5-component model. For the conditions of the motion along the rebounding trajectory, the ionization degree attains 20% in the 11-component model. Nevertheless, the difference between the 7 and 11-component models is observed only over the starting trajectory section and does not exceed 10% for the heat flux and 20 K for the equilibrium surface temperature. When the 5 and 7-component models are used the difference in the values of the heat flux and surface temperature is practically absent, and it is graphically shown in Fig. 1.

Heat transfer is, to a great extent, affected by vibrational relaxation rather than by ionization. The calculations made, assuming the equilibrium excitation of all internal degrees of freedom (curve 3, Fig. 1), can underestimate by almost twice the values of the heat flux at heights of more than 90 km. At the same time, over the most heat-stressed trajectory section at $H \lesssim 85$ km the difference in the heat flux values does not exceed 2%. This can be attributed to an inconsiderable concentration of molecules because of strongly developed dissociation.

The vibrational relaxation effects also result in a substantial increase of the translational temperature (Fig. 2), thus yielding the flow rarefaction in the shock layer and increasing the size of the disturbed flow region. The dissociation degree in this case decreases (Fig. 3) due to the presence of a multiplier V in expression (4) for the for the dissociation rate constant.

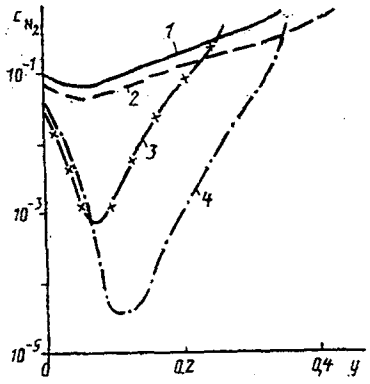

Fig. 3

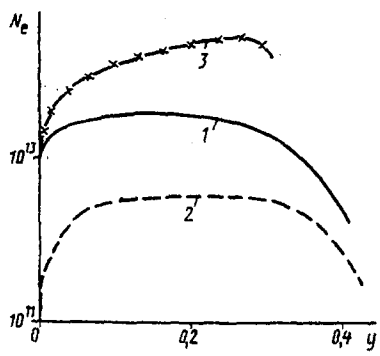

Fig. 4

Fig. 3. N_2 mass concentration profiles at H = 86 km, V_∞ = 9.8 km/sec. The notation is the same as in Figs. 1 and 2.

Fig. 4. Electron concentration profiles at H = 88 km, V_∞ = 7.66 km/sec. The notation is the same as in Fig. 1. N_e, cm^{-3}.

Ionization strongly affects the profiles of the unknown quantities. When replacing the 5 and 7-component models by the 11-component one a 20% decrease in the shock layer thickness occurs (H = 86 km, V_∞ = 9.8 km/sec). The 11-component model is characteristic of a more sharp decrease of the translational temperature behind the leading front of the shock wave.

The electron concentration in the shock layer is the most important characteristic. It is the subject of the problems of radio communication, detection, etc. Figure 4 illustrates the need to allow for the nonequilibrium excitation of vibrational degrees of freedom and to use the 11-component air model for calculations of electron concentration profiles.

Thus, these results show that for the conditions of the body motion along the rebounding trajectory, the difference between the 5, 7, and 11-component air models does not exceed 10% in the heat flux values. At the same time, the number of ionized components that are taken into account greatly influences the shock layer structure and can result in the difference of the electron concentration by an order of magnitude. Account of the vibrational relaxation effects can increase the heat flux values up to twice and decrease the electron concentration value by an order of magnitude.

NOTATION

y, distance along the axis from a body; xu and v, physical velocity vector components in the x and p-directions; p, ρ, T, T_v, pressure, density, translational, and vibrational temperatures, respectively; μ, λ^{tr}, λ^v, viscosity coefficient, thermal conductivities; x_i, c_i, m_i, h_i, c_{pi}, I_i, V_i, $\dot{w}_i$, e_i, molar concentration, mass concentration, molecular mass, enthalpy, heat capacity, normal vector component of the diffusional flow, normal vector component of the diffusional velocity, rate of forming a mass, i-th component charge; e_k, θ_{kv}, specific vibrational energy, characteristic vibrational temperature of the k-th molecule; Q_k, source terms; E, normal vector component of the electric field strength; k, e, Planck constant, electron charge; R_A, universal gas constant; S_{ij}, Schmidt number; N, number of components (including electrons); M, number of molecular components. Indices: ∞, w, parameters in the incoming flow and on the body surface; tr and v, translational–rotational and vibrational degrees of freedom; i = H, summation over heavy particles (including electrons), i = M, only over molecules.

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RESTRICTIONS IN IMPLEMENTATION OF MULTISTAGE LASER AMPLIFIERS

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The results of a simultaneous account of the influence of the amplifying properties of an active medium of a laser amplifier and its inhomogeneities on energy characteristics in an application zone are given. An intensity is described to the approximation of linear and saturated gain. Distortions in the active medium are accounted for by the wave front dispersion. A simultaneous account of inverse indices and optical quality has allowed determination of an optimal number of amplifying stages of a laser unit providing the maximum radiation power density in the application zone.

The most workable arrangement for the design of high-power lasers is a master oscillator–amplifier combination [1]. However the requirement of optical homogeneity of an active medium restricts the amplifier length along a light propagation path. Therefore it is necessary to simultaneously analyze the influence of inverse characteristics of an active medium and the degree of its integrated inhomogeneities on energy indices of laser radiation in the application zone.

In the absence of resonance-free losses in an active medium the light intensity in an amplifier laser may be described [2] as $I = I_0 \exp(kx)$ for the linear gain and $I = I_0 + kx$ for the saturated gain. Inhomogeneities in an active medium exert an influence on the phase characteristics of radiation [1]. When light passes through the active medium with a length L_1 and a wave front undergoes distortions characterized by dispersion D [3], the energy in the central lobe of the radiation directivity diagram in the Fraunhofer diffraction zone is determined as [3]

$$W = \frac{W_{-1}^{+1}}{W_{0\ -1}^{+1}} = \mathrm{Sh} = \exp(-D),$$

where W_{-1}^{+1} is the power fraction in the central lobe of the directivity diagram for radiation diffraction with a distorted wave front; $W_{0\ -1}^{+1}$ is the power fraction in the central lobe of the directivity diagram for radiation diffraction with a plane wave front (for a square aperture $W_{0\ -1}^{+1} = 0.81$, for a circular one, 0.84); Sh is the Strel number [4].

A single-pass amplifier with length L_1 with k and D for linear gain gives the following power in the far-field zone:

$$W_1 = WSI = SI_0 \exp(kL_1 - D).$$

In order to increase the output power of a laser unit with a preassigned gain of an active medium, it is necessary to increase the amplifier length along the beam or to provide a tandem (multistage) circuit including the necessary number of modules-amplifiers. If the number of stages with the length L_1 is n, then the relative power at the amplifier outlet is

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